STABILITY OF CYLINDRICAL SHELLS SUBJECTED TO COMBINED AXIAL COMPRESSION AND INTERNAL PRESSURE UNDER CREEP CONDITIONS

L. M. Kurshin and V. T. Shcherbakov

UDC 629.735.33.023.2

The critical strain of a cylindrical shell subjected to combined axial compression and internal pressure is computed under creep conditions. A method is proposed to determine values of the initial deflections by means of elastic shell test data for a creep analysis of shells. Data of an experimental investigation of the creep stability of shells are presented, which are compared with the results of the computation.

1. Certain values of the initial imperfections of the middle surface should be given for the stability analysis of a cylindrical shell under creep conditions in conformity with the method elucidated in detail in [1]. The method of selecting the initial deflections for an analysis of a compressed cylindrical shell was examined in [2], where it has been shown that combinations of values of the symmetric and nonsymmetric compounds of the initial deflection can be selected from the data of an elastic experiment, and small symmetric and large nonsymmetric deflections should be taken to compute the strain under creep conditions.

The influence of the internal pressure on the critical strain during shell compression under creep conditions is investigated in this paper.

To solve the problem of creep stability of a cylindrical shell subjected to axial compressive forces and internal pressure, it is necessary to have the solution of the elastic problem for a shell with an initial deflection as the initial condition. Letting Φ and w denote the stress and deflection functions, respectively, let us write the nonlinear equations of a shallow cylindrical shell

$$(1/B)\,\Delta\Delta\Phi = (1/R)(w - w_0)_{xx} + w_{xy}^2 - w_{xx}w_{yy} - (w_{0,xy}^2 - w_{0,xx}w_{0,yy})$$
$$-D\Delta\Delta(w - w_0) - (1/R)\,\Phi_{xx} + \Phi_{yy}w_{xx} + \Phi_{xx}w_{yy} - 2\Phi_{xy}w_{xy} + q = 0 \qquad (1.1)$$

Here $B = Eh$, $D = (^8/_9)Eh^3$, E is the elastic modulus, 2h and R are the shell thickness and radius, respectively.

Let us give the initial deflection in the form

$$w_0 = f_1^0 \sin(^1/_2)\,\alpha x \sin(my/R) + f_2^0 \cos\alpha x \qquad (1.2)$$

Assuming the bending mode to be conserved under loading, we take

$$w = f_1 \sin(^1/_2)\,\alpha x \sin(my/R) + f_2 \cos\alpha x + f_3 \qquad (1.3)$$

Having determined the function Φ from the first equation in (1.1) and having integrated the second with respect to the coordinates x and y in the Bubnov–Galerkin sense, we obtained a system of nonlinear equations in the dimensionless elastic deflections $\zeta_1 = f_1$ 2h and $\zeta_2 = f_2/2h$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 109–116, September–October, 1974. Original article submitted June 14, 1973.

$$\gamma_1 \zeta_1^3 + n_1 \zeta_1 + n_2 \zeta_1^0 = 0 \tag{1.4}$$

$$\{(^{16}/_{9})\,\vartheta^2 - 1\,/\,(4\vartheta^2\eta^2)\,[(\eta\,/\,4)\,(\zeta_1^2 - \zeta_1^{02}) + \zeta_2^0]$$
$$+ (2\,/\,\vartheta^2)\,\zeta_1\zeta_1^0\zeta_2^0\gamma_3 + [\zeta_1\,(\zeta_1 - \zeta_1^0)]\,/\,(2\eta\vartheta^2\lambda_1^2)\}\,/\,n_3\zeta_2 = 0$$

$$n_2 = (4\,/\,\eta\lambda_1^2)\,(\zeta_2 + \zeta_2^0) - \gamma_2 - 16\zeta_2\zeta_2^0\gamma_3$$

$$n_3 = (^{16}/_{9})\,\vartheta^2 - (4p)\,/\,(3\eta) + 1\,/\,(4\vartheta^2\eta^2) + (2\zeta_1^2\gamma_3)\,/\,\vartheta^2$$
$$\gamma_1 = (\vartheta^2 + 1)\,/\,4 \;\; \vartheta_2 = (^4/_9)\,\vartheta^4\lambda_1^2 + 1\,/\,(\eta^2\lambda_1^2)$$
$$\gamma_3 = 1\,/\,\lambda_1^2 + 1\,/\,(81\lambda_2^2), \quad \lambda_1 = (\vartheta^2 + 1)\,/\,\vartheta^2$$
$$\lambda_2 = (9\vartheta^2 + 1)\,/\,(9\vartheta^2)$$

Here we have introduced the quantities

$$\zeta_1^0 = f_1^0\,/\,2h, \;\; \zeta_2^0 = f_2^0\,/\,2h, \;\; \vartheta = (\alpha R)\,/\,(2m), \;\; \eta = (m^2h)\,/\,R$$

as well as the dimensionless parameters of the axial compression p and the internal pressure q,

$$p = (3R\sigma)\,/\,(4Eh), \quad q = (q^* R^2)\,/\,(2Eh^2)$$

where σ, q^* are the axial compressive stress and the internal pressure.

Equations (1.4) determine the dependences of the deflections ζ_1 and ζ_2 on the loads p and q. The determinant

$$M = a_{11}a_{22} - a_{12}a_{21} \tag{1.5}$$

vanishes when the limit points on the strain curve are reached.

In the case of large initial deflections, the quantity M does not vanish, and the condition that the minimum value of (1.5) is reached, which corresponds to the maximum strain rate of the shell, is used as the stability criterion.

We have used the following notation in (1.5):

$$a_{11} = 2\gamma_1\zeta_1^2 + n_1, \quad a_{12} = n_3$$
$$a_{21} = 32\gamma_3\zeta_1\zeta_2 - (1\,/\,\eta)\,(8\,/\,\lambda_1^2 + 1) + (4\zeta_1^0)\,/\,(\eta\lambda_1^2) - 16\gamma_3\zeta_1^0\zeta_2^0$$

$$a_{22} = (4\,/\,\vartheta^2)\,\gamma_3\zeta_1\zeta_2 - (1\,/\,\eta)\,[1\,/\,(\vartheta^2\lambda_1^2) + 1\,/\,(8\vartheta^2)]\,\zeta_1 - (2\,/\,\vartheta^2)\,\zeta_1^0\zeta_2^0\gamma_3 + \zeta_1^0\,/\,(2\eta\vartheta^2\lambda_1^2)$$

Presented in Fig. 1 are the dependences of the symmetric (dashed curves) ζ_2 and nonsymmetric (solid curves) ζ_1 deflection on the compressive load p according to (1.4) for various values of the internal pressure parameter q = 0, 0.2, 0.4, 0.6, 0.8, 1.0 (curves 1-6, respectively) for $\zeta_0 = \zeta_{0k} = 0.2$, $\zeta_0 = 3\zeta_2^0$, $\zeta_{0k} = 3\zeta_1^0$. The following wave-formation parameters were taken in the computations: $\vartheta = 1$ (related to the shape

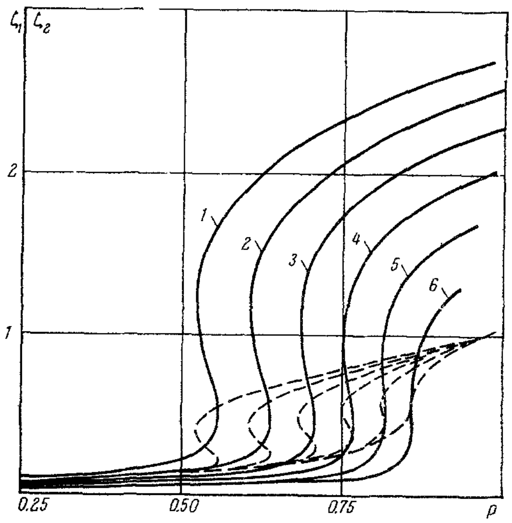

Fig. 1

of the dents), $\eta = 0.375$ (characterizes the quantity of dents over the circumference). The value $\eta = 0.375$ of the parameter corresponds to the buckling mode of an ideal shell under pure compression, and the value $\vartheta = 1$ corresponds to square nonsymmetric dents.

Computations for the selection of these parameters from the condition of a minimum critical load (or the critical time under creep conditions) permit making the deduction that the results depend slightly on the values of these parameters in a known range. For the compression case, the results of such computations are presented in Fig. 17 of [1], for example. The parameters ϑ and η are related to the wave-formation parameters in [1] as follows:

$$\vartheta = \theta, \qquad \eta = (^3/_8)\beta^{*2}$$

Presented as an illustration in Fig. 2 are dependences of the critical axial compression load for diverse combinations of the initial deflections. The values of the initial deflections ζ_0 and ζ_{0k} are given below for curves 1-12:

No.	1	2	3	4	5	6	7	8	9	10	11	12
ζ_0	0.01	0.01	0.01	0.05	0.1	0.1	0.2	0.2	0.2	0.6	0.6	0.6
ζ_{0k}	0.05	0.1	0.2	0.2	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2

Various combinations of the symmetric ζ_0 and nonsymmetric ζ_{0k} components of the initial deflection correspond to the very same critical value of the compressive load p. A number of such combinations is presented below for the case p = 0.5. The dependences of p on q for these initial deflection combinations will appear as a fan of curves (Fig. 3) when computing the critical values

No.	1	2	3	4	5	6	7	8
ζ_0	0	0.09	0.165	0.26	0.39	0.46	0.50	0.51
ζ_{0k}	1.0	0.8	0.6	0.4	0.15	0.06	0.015	0

As a comparison, the dashed lines show the curves from [3], at which the critical load was sought when terms associated with discarding deflection amplitudes of more than second degree in the nonlinear equations of the type (1.4). The curve *a* corresponds to a nonsymmetric initial deflection ($\zeta_0 = 0$), while curve b corresponds to the symmetric deflection ($\zeta_{0k} = 0$). It is seen that this simplification exaggerates the critical axial compression load for a nonsymmetric initial deflection.

2. Let us use the equations in [4] which take account of a geometric nonlinearity for the perturbed state of a cylindrical shell with an initial deflection under creep conditions. For the case of power-law creep $p_i = A\sigma_i^n$ (where p_i, σ_i are the creep rate and stress intensities) and axial compression and internal pressure acting on the shell, the equations are

$$\Delta\Delta\Phi + (n-1)\, e^{-\xi}\int_0^{\xi} e^{\xi}\Lambda_1\Lambda_1\Phi\, d\xi - B\left[\Gamma(w, w_0) - e^{-\xi}\int_0^{\xi} e^{\xi}\Gamma(w, w_0)\, d\xi\right] = 0 \tag{2.1}$$

$$U(w, w_0, \Phi) + e^{-\xi}\int_0^{\xi} e^{\xi} D(\Delta\Delta - {}^3/_4\,\Lambda\Lambda)(w - w_0)\, d\xi + (3n/4)\, De^{-n\xi}\int_0^{\xi} e^{n\xi}\Lambda\Lambda(w - w_0)\, d\xi + q = 0$$

$$\Gamma(w, w_0) = w_{xy}{}^2 - w_{xx}w_{yy} - (w_{0,\,xy}{}^2 - w_{0,\,xx}w_{0,\,yy}) + (1/R)(w - w_0)_{xx}$$

$$U(w, w_0, \Phi) = -D\Delta\Delta(w - w_0) - (1/R)(q^*R + \Phi_{xx}) + 2h\sigma_i\Lambda w + \Phi_{yy}w_{xx} + \Phi_{xx}w_{yy} - 2\Phi_{xy}w_{xy}$$

$$\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2, \qquad \Lambda = -(1/\varkappa)\,\partial^2/\partial x^2 + (\lambda/\varkappa)\,\partial^2/\partial y^2$$

$$\Lambda_1 = -(1/\varkappa)\,\partial^2/\partial y^2 + (1/2\varkappa)\,\partial^2/\partial x^2 + (\lambda/\varkappa)\,\partial^2/\partial x^2 - (\lambda/2\varkappa)\,\partial^2/\partial y^2$$

$$\varkappa = \sqrt{1 + \lambda + \lambda^2}, \qquad \lambda = (q^*R)/(2h\sigma) = (3q)/(4p)$$

$$\sigma_{11} = -\sigma, \qquad \sigma_{22} = (q^*R)/(2h),$$
$$\xi = (Ep_i)/\sigma_i = EA\sigma_i^{n-1}t$$

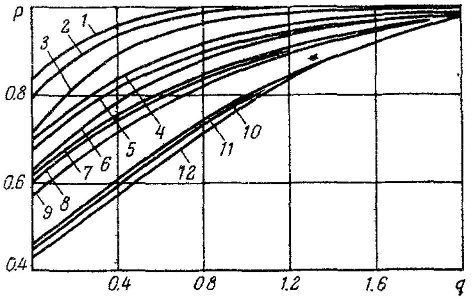

Fig. 2

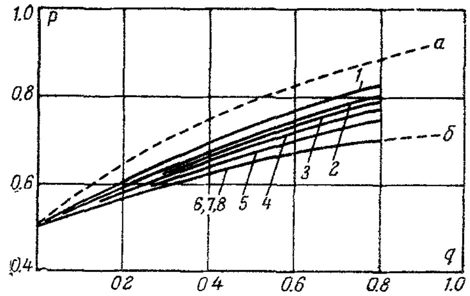

Fig. 3

Let us seek the solution of (2.1) in the form

$$w = \varphi_1(\xi) f_1 \sin(1/2)\alpha x \sin(my/R) + \varphi_2(\xi) f_2 \cos\alpha x + \varphi_3(\xi) f_3 \tag{2.2}$$

$$\Phi = \psi_1(\xi) A_1 \cos\alpha x + \psi_2(\xi) A_2 \cos(my/R) +$$

$$+ \psi_3(\xi) A_3 \sin(1/2)\alpha x \sin(my/R) + \psi_4(\xi) A_4 \sin(3/2)\alpha x \sin(my/R)$$

Here f_1, f_2 are the solution of the system (1.4) corresponding to elastic strain of the shell. We have the following initial conditions,

$$\varphi_1(0) = \varphi_2(0) = \varphi_3(0) = \psi_1(0) = \psi_2(0) = \psi_3(0) = \psi_4(0) = 1 \tag{2.3}$$

at $\xi = 0$ for Eqs. (2.1) describing the creep process.

Integrating (2.1) with respect to the coordinates x and y in the Bubnov–Galerkin sense, we obtain a system of nonlinear integral equations for the functions $\varphi_i(\xi)$

$$a_1\varphi_1 + a_2\varphi_2 + a_3 = 0, \qquad b_1\varphi_1 + b_2\varphi_2 + b_3 = 0 \tag{2.4}$$

$$a_1 = -\zeta_1[g_1 - (4/3)p - (\eta/4) g_2 \zeta_1^{02} + (1/\vartheta^2)\zeta_2^0 + q/\vartheta^2]$$

$$a_2 = [(4\zeta_1^0\zeta_2)/\vartheta^2] g_3$$

$$a_3 = -\zeta_1^0[g_1 - (4\zeta_2^0)/(\vartheta^2\lambda_1^2)] - (1/\vartheta^2) g_4\varphi_1\varphi_2\zeta_1\zeta_2 +$$

$$+ (\eta/4) g_2\varphi_1^3\zeta_1^3 + (16\eta/\vartheta^2)\gamma_3\varphi_1\varphi_2^2\zeta_1\zeta_2^2 - (k_2/4) g_8\varphi_1\zeta_1 J_{22} +$$

$$+ (2k_3/\vartheta^2\lambda_1^2)[2\varphi_2\zeta_2 - 1/(2\eta)] g_5 + (k_1/\vartheta^2) g_6\varphi_1\zeta_1 -$$

$$- (16k_4\eta)/(81\vartheta^2\lambda_2^2)\varphi_2\zeta_2 J_{44} - (g_8/3)[(4/3)\lambda_1^2 -$$

$$- \lambda_3^2/\varkappa^2] J_{53} - [(ng_8)/3](\lambda_3^2/\varkappa^2) J_{23}$$

$$b_1 = (2\zeta_1^0\zeta_1 g_3)/\vartheta^2, \qquad b_2 = \zeta_2[g_7 - (16/3)p]$$

$$b_3 = -g_7\zeta_2^0 + \zeta_1^{02}/(4\vartheta^2) - (g_4\varphi_1^2\zeta_1^2)/(4\vartheta^2) +$$

$$+ (8\eta\gamma_3\varphi_1^2\varphi_2\zeta_1^2\zeta_2)/\vartheta^2 + (2k_3\varphi_1\zeta_1 g_5)/(\vartheta^2\lambda_1^2) - (k_1 g_6)/g_8 -$$

$$- (8\eta k_4\varphi_1\zeta_1 J_{44})/(81\vartheta^2\lambda_2^2) - (16g_8/9)[(4 - 3/\varkappa^2) J_{51}$$

$$+ (3nJ_{21})/\varkappa^2]$$

$$g_1 = (4\vartheta^2\eta\lambda_1^2)/9 + 1/(\vartheta^2\eta\lambda_1^2), \qquad g_2 = \vartheta^2 + 1/\vartheta^2$$

$$g_3 = 1/\lambda_1^2 - 4\eta\zeta_2^0\gamma_3, \qquad g_4 - 1 + 8/\lambda_1^2$$

$$g_5 = J_{33} - 4\eta J_{34}, \qquad g_6 = J_{11} - (\eta/4) J_{12}$$

$$g_7 = (64\vartheta^2\eta)/9 + 1/(\vartheta^2\eta), \qquad g_8 = \vartheta^2\eta$$

$$\lambda_3 = (\vartheta^2 - \lambda)/\vartheta^2, \quad J_{ij} = e^{-k_i\xi}\int_0^\xi e^{k_i\xi} H_j(\xi)\,d\xi$$

$$H_1 = \varphi_2\zeta_2 - \zeta_2^0, \qquad H_2 = \varphi_1^2\zeta_1^2 - \zeta_1^{02}$$

$$H_3 = \varphi_1\zeta_1 - \zeta_1^0, \qquad H_4 = \varphi_1\varphi_2\zeta_1\zeta_2 - \zeta_1^0\zeta_2^0$$

$$k_1 = 1 + (n-1)(1/2 + \lambda)^2/\varkappa^2$$

$$k_2 = 1 + (n-1)(1 + \lambda/2)2/\varkappa^2$$

$$k_3 = 1 + (n-1)([1/2 + \lambda)\vartheta^2 - (1 + \lambda/2)]^2/[\varkappa^2(\vartheta^2 + 1)^2]$$

$$k_4 = 1 + (n-1)[(1/2 + \lambda) 9\vartheta^2 - (1 + \lambda/2)]^2/[\varkappa^2(9\vartheta^2 + 1)^2]$$

Integrating the system (2.4) with respect to ξ, we find an expression for the determinant:

$$M = a_{11}a_{22} - a_{12}a_{21} \tag{2.5}$$

whose zero value corresponds to a limit point on the deflection curve during creep and determines the dimensionless critical parameter $\xi_k = EA\sigma_i^{n-1}t$ related to the time. The values of the deflections φ_1, φ_2 are obtained by numerical integration of (2.4) as a function of the dimensionless time. Shown as an illustration

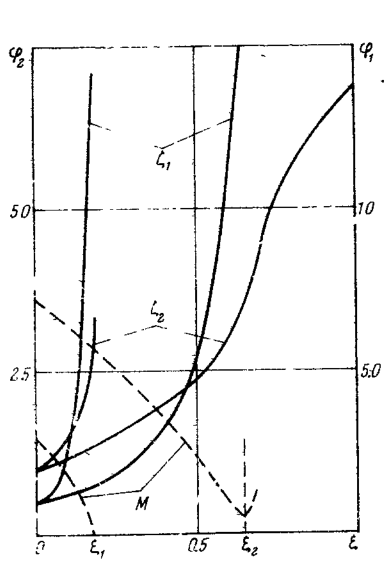

Fig. 4

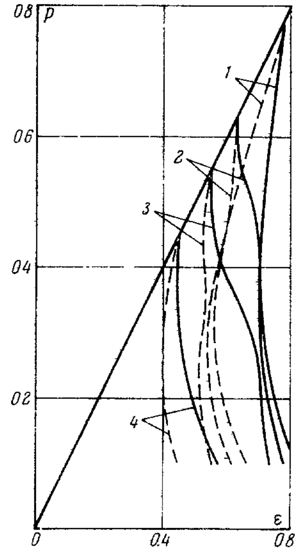

Fig. 5

in Fig. 4 are the dependences of the symmetric φ_2 (ξ) and nonsymmetric φ_1 (ξ) deflections for two values of the axial load p and the same value of the pressure q = 0.075 (ζ_0 = 0.2, ζ_{0k} = 0.05, n = 3). The dashed lines show values of the determinant M (ξ_1 corresponds to the limit and ξ_2, to the inflection points).

On the basis of the computations performed, it can be concluded that both the symmetric and the nonsymmetric deflections grow during creep under large axial compression loads. As the axial load diminishes at the same internal pressure, the nonsymmetric deflection grows more intensively than does the symmetric deflection. The shell buckles in an axisymmetric mode. The shell buckles in a nonaxisymmetric mode up to some ratio between the internal-pressure parameter q and the compressive-load parameter p, then the shell buckles in a symmetric mode as this parameter increases. This agrees qualitatively with the results of experimental investigations [5].

The total critical strain ξ_k comprised from the elastic strain and the shell strain accumulated up to the time of buckling under creep conditions,

$$\varepsilon = p\,(1 + \xi_k) \tag{2.6}$$

was calculated by means of the value of the critical parameter ε.

Results of computing the critical dimensionless axial strain ε (2.6) as a function of the axial load obtained by solving (2.4) by using the criterion (2.5) for different values of the initial pressure parameter q = 1, 0.5, 0.25, 0.0 (curves 1-4, respectively), are presented in Fig. 5. The results of the computations correspond to ζ_0 = 0.5, ζ_{0k} = 0.2. The solid curves refer to n = 3 and the dashed curves, to n = 7.

As the internal pressure increases to the value q = 0.25, the critical strain grows for all the values of the compressive stress p. As the internal pressure increases further, the critical strain grows only for large compression stresses. The inflections on the critical strain curves are related to the replacement of the nonsymmetric by the symmetric buckling mode. As the creep index n increases, the critical strains diminish for the very same level of p and q.

3. An experimental investigation of the stability of cylindrical shells under creep was conducted to verify the computed results. Shells turned from the material D16T were tested; the geometric shell dimensions were: thickness 2h = 0.5 mm, radius R = 88 mm, length l = 425 mm. The test temperature was T = 250°C. The deviation of the temperature along the shell generator and over the circumference did not exceed 5°C. The axial load, the time, and contraction of the shell by which the creep strain was estimated were measured in the experiment. A total of 13 shells were tested for stability under axial compression and internal pressure without heating. Also, two shells were tested at the same internal pressure. The results of the elastic tests are represented by points in Fig. 6. The critical compressive load grows as the pressure rises in the investigated pressure range.

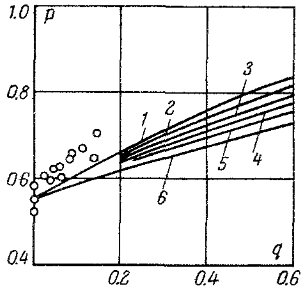

Fig. 6

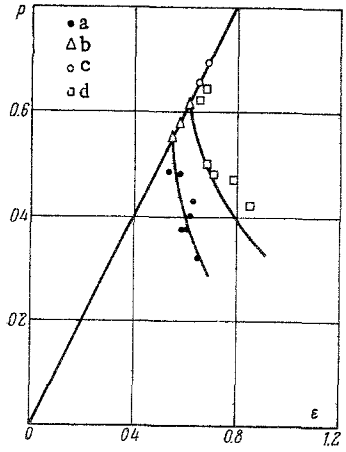

Fig. 7

In all, ten shells were tested for stability under creep conditions and compression without internal pressure. The test results are presented in Fig. 7 (a and b). A total of seven shells (a) were tested under different constant compressive load levels; three shells (b) were tested without creep under rapid (5-10 sec) loading up to buckling (T = 250°C).

We tested eight shells for stability under creep with axial compression and the internal pressure $q^* = 1\ kg/cm^2$ ($q = 0.11$). After the shell had been heated to T = 250°C, an internal pressure was supplied, then the axial load was applied. Then two shells (c) were loaded rapidly (5-10 sec) up to buckling, the rest (d) were tested for creep with axial loads comprising a part of the load for buckling. It is seen that internal pressure increases the critical creep strain considerably.

The results obtained permit the expression of reasoning about the method of the stability analysis of shells subjected to axial compression and internal pressure under creep conditions. The main difficulty is the correct selection of the values of the initial deflection components (the symmetric ζ_0 and nonsymmetric ζ_{0k}) introduced in the computation.

If there are shell test results on the elastic stability under compression without internal pressure, then a combination of values of the initial deflections can be determined. Having the elastic test data with internal pressure and axial compression available, a combination of the symmetric ζ_0 and nonsymmetric ζ_{0k} initial deflections, corresponding best to the data of the elastic experiment with internal pressure, can be found from the set of combinations of the initial deflections. The deflections selected in this manner are then inserted in the computation of shells operating under creep conditions with a combined loading.

Let us determine the initial deflections and then the critical strains according to the proposed method for shells whose creep test results have been presented above.

The magnitude of the critical load under elastic buckling without internal pressure for the tested shells is $p = 0.55$ (Fig. 6). The combinations of initial deflections presented below, which have been selected from the results of solving the elastic problem of axial compression of a cylindrical shell [1], correspond to this critical load. Dependences of the critical axial compression load on the internal pressure have been obtained for these combinations of the initial deflections by means of (1.4). A series of such curves obtained by computations is presented in Fig. 6 and corresponds to combinations of initial deflections (see below).

No.	1	2	3	4	5	6
ζ_0	0	0.03	0.075	0.12	0.22	0.35
ζ_{0k}	0.68	0.60	0.51	0.40	0.20	0.03

The initial deflections $\zeta_0 = 0$, $\zeta_{0k} = 0.68$ correspond best to the shells tested (Fig. 6). The dependences of the total critical shell strain under creep, shown by curves 1, 2 in Fig. 7 for the cases $q = 0$ and 0.11, respectively, which were computed by means of (2.4)-(2.6), correspond to these values of the initial deflections.

The agreement between the results of analyzing shells under creep by the method used and the results of the experiment is satisfactory.

LITERATURE CITED

1. L. M. Kurshin, "Stability under creep conditions," Analysis of Spatial Structures [in Russian], No. 13, Stroiizdat, Moscow (1970).
2. L. M. Kurshin and V. T. Shcherbakov, "On the stability analysis of cylindrical shells under compression in creep conditions," Problems of the Structural Mechanics of Ships, [in Russian], Sudostroenie, Leningrad (1973).
3. J. Hutchinson, "Axial buckling of pressurized imperfect cylindrical shells," AIAA J., 3, No. 8 (1965).
4. L. M. Kurshin, "On the formulation of the problem of creep buckling of shells," Dokl. Akad. Nauk SSSR, 163, No. 1 (1965).
5. P. M. Ogibalov and V. F. Gribanov, Thermostability of Plates and Shells, [in Russian], Izd. MGU, Moscow (1968).